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A Thomistic Reply to Grünbaum's Critique of Maritain on the Reality of Space

I. Introduction

This essay reviews two critiques of Jacques Maritain's account of the reality of space. One is Adolf Grünbaum's, the other is Charles De Koninck's. The goal is to adjudicate between these differing views in the philosophy of space. The dispute concerns the existence of the metric of physical space and how it is discovered.

This implicates deep questions about the being and intelligibility of physical quantity and consequently the possibility of knowing the universe in general. This is not to overstate the case, for the failure of physico-mathematical secularism warrants such a claim. This secularism tries to settle an ancient question, which we could call *the central philosophical question* about mathematical physics: "Are mathematical objects different in some fundamental way from physical objects?"¹ Famously, both Platonists and Aristotelian-Thomists give affirmative

¹ Richard F. Hassing, "Modern Turns in Mathematics and Physics," in *The Modern Turn*, edited by Michael Rohlf, vol. 60, Studies in Philosophy and the History of Philosophy (Washington, D.C.: The Catholic University of America Press, 2017), 169.

answers, though for markedly different reasons. The outright Cartesian negative answer is nuanced by the Newtonian approach: any possible difference between the mathematical and the physical could be ignored and mathematical physics could still make progress. This is physico-mathematical secularism. Just as political secularism considers the metaphysical roots of moral and religious principles to be private affairs barred from dictating civic policy, so also physico-mathematical secularism makes the ancient metaphysical question a private philosophical concern, not to be mixed with public scientific practice. This assumption of the irrelevance of the ancient central question governed the practice of classical physics (pre-quantum and pre-relativistic) because algebraic vector quantities as mathematical objects permitted the infinitely precise spatiotemporal imageability of fundamental physical processes and physical objects.² Thus, the question of the relationship between vector quantities as mentally existing versus how they might exist in things was rendered moot by the aforementioned secularism, even as the adequacy of such a relationship—that the mathematical mode *adequately matches* the physical mode—was assumed.

However, the demise of classical physics destroyed this secularism: “The Heisenberg uncertainty principle is emblematic of the failure of physico-mathematical secularism.”³ In analogous fashion, the development of non-Euclidean geometries motivated doubts concerning the presumptive Euclidean character of physical space.⁴ This concern with regard to the reality of

² Ibid., 143.

³ Ibid., 174.

⁴ Hans Reichenbach, *The Philosophy of Space and Time*, translated by Maria Reichenbach and John Freund (New York: Dover Publications, 1957), 35–36.

physical objects insofar as they are known through mathematical ones can be illustrated by the following critique of Grover Maxwell's claim about the key geometric object of general relativity, the metric tensor that captures the geometry of a gravitational field: "If we were carrying a heavy suitcase in a changing gravitational field, we could observe the changes of the $G_{\mu\nu}$ of the metric tensor."⁵ William A. Wallace offers the following criticism:

There is considerable difference between being aware of a body's gravity or heaviness

⁵ A tensor is a generalized algebraic quantity (not an Aristotelian categorical, quantity). More familiar are tensors of rank-zero, *scalar* quantities like temperature, or rank-one tensors, *vector* quantities like velocity: "Tensors are generalizations of scalars (that have no indices), vectors (that have exactly one index), and matrices (that have exactly two indices) to an arbitrary number of indices," where an "index" refers to the dimensions of the space represented (e.g., Euclidean space); Todd Rowland and Eric W. Weisstein, "Tensor," *MathWorld—A Wolfram Web Resource*, URL: <<https://mathworld.wolfram.com/Tensor.html>>, accessed 6-25-2020. The metric tensor is the tensor representing measured geometric structure; thus, it allows one to calculate distance. In general relativity, the metric tensor measures the curvature of spacetime. If massive bodies "bend" spacetime, that is, if space tells matter (mass-energy) how to move, and matter tells space how to curve, the metric tensor does the telling on behalf of space, while the stress-energy tensor does the telling on behalf of matter; see Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler, *Gravitation* (Princeton, NJ: Princeton University Press, 2017), 5. So, Maxwell is claiming that an action we can perform and sense allows us to "observe" the characteristic differences in what is *prima facie* an abstract algebraic object.

and of ‘directly observing’ a gravitational ‘field’ or a ‘metric tensor.’ If this is so, *the cause of realism is not served by assigning equivalent ontological status to physical attributes and to theoretical constructs used to calculate the metrical aspects of such attributes.* Nor can one easily assign to the so-called elementary particles of modern physics a degree of reality that would place them on a par with the tables and chairs of ordinary experience.⁶

Therefore, just as the difference between mathematical and physical objects is crucial for understanding the nature of fundamental micro-processes of matter, so also is this difference crucial for understanding the nature of the universe on the largest scales of spatial behavior and structural development. Consequently, any weakness in explicating the true character of this difference will translate into a weakness in explicating one’s cosmological ontology.

Accepting this theoretical maxim, the argument proceeds as follows. First, we review Grünbaum’s criticisms of Maritain (section 2), and see to what extent they engage with Maritain’s views (section 3). Then, De Koninck’s arguments are summarized (section 4), explained (sections 5–6), and his refutation compared with Grünbaum’s (section 7). We then adjudicate between Grünbaum, Maritain, and De Koninck (sections 8–9). In conclusion (section 10), De Koninck’s criticisms of Maritain not only meet his opponent’s positions more precisely by way of refutation than do Grünbaum’s, but their Thomistic background is more promising by way of an ontology adequate to the cosmos.

II. Grünbaum’s Critique

⁶ William A. Wallace, review of *Philosophical Problems of Space and Time*, by Adolph Grünbaum, *The Thomist* 28, no. 4 (1964): 525–26; emphasis added.

The backdrop for Grünbaum's critique of Maritain is a general critique of the Duhemian character of Einstein's philosophy of geometry.⁷ That is, Grünbaum maintains that it is possible to determine an empirical measurement procedure that would discover whether or not the physical space of the universe is Euclidean or not. By contrast, the Duhemian view about the geometry of physical space is an instance of the general Duhem-Quine thesis that one cannot refute a scientific hypothesis in isolation from its various auxiliary hypotheses. In this case, the auxiliary hypotheses could be modified to always allow for one to undermine an empirical claim to have determined the actual geometry of physical space. This is what Grünbaum denies by elaborating an empirical procedure to test the geometry of physical space.⁸

Grünbaum then wonders what would happen if his procedure, or other scientific replacements of it, failed: "[T]hen, it seems to me, we would unflinchingly have to resign ourselves to this relatively unmitigable type of uncertainty. No, says the philosopher Jacques Maritain."⁹ Why? Because Maritain claims *qua* philosopher that the metric of real space is Euclidean. Grünbaum chooses to rebut Maritain's views "because they typify the conception of those who believe that the philosopher as such has at his disposal means for fathoming the structure of external reality which are not available to the scientist."¹⁰ Grünbaum notes Maritain's Duhemian rejection of the possibility of physical measurement uniquely determining

⁷ Adolf Grünbaum, *Philosophical Problems of Space and Time*, 2nd ed. (Dordrecht: Springer, 1973), 106–47.

⁸ See *ibid.*, 144–47.

⁹ *Ibid.*, 147.

¹⁰ *Ibid.*, 148.

the metric of real space (for measuring instruments must presume a geometry in their construction and use). Grünbaum then considers a destructive dilemma posed by Maritain that attempts to manifest Euclidean geometry as the only geometry of real space. Grünbaum glosses this argument thus: non-Euclidean geometries depend upon the Euclidean case both for their logical consistency and their intuitability. Thus, non-Euclidean geometries can be mental objects—beings of reason—but they cannot characterize the real space which both intuition and logic apprehends to be Euclidean. (We examine this dilemma in more detail below.) Grünbaum submits: “Maritain’s thesis is unsound in its entirety and can be completely refuted.”¹¹ His first three counter-arguments are based upon the logical self-consistency of the various geometries, demonstrated by Klein and Hilbert, and upon the intrinsic properties of non-Euclidean spaces when not embedded in a Euclidean space as a model. These three arguments, as they occlude deep differences between the foundations of modern mathematics and Maritain’s Thomistic approach, will not be examined in detail here.

It is instead Grünbaum’s last argument that is paramount. Maritain errs, he claims, regarding what it means for existing bodies to have geometric properties. This is because Maritain’s account of the mind’s activity in constituting geometric objects mischaracterizes the act of abstraction.

It can surely not be maintained that “the geometric properties of existing bodies” are “those properties which the mind recognizes in the elimination of all the physical.”¹² For,

¹¹ Ibid., 150.

¹² Grünbaum quotes Maritain’s *The Degrees of Knowledge* as translated by Bernard Wall (London: G. Bles, The Centenary Press, 1937), 204. Wallace in “Review,” 529, surmises that

in that case, geometry would be the study of purely imagined thought-objects, which will, of course, turn out to have Euclidean properties, if Maritain's imagination thus endows them. And the geometry of such an imagined space could then not qualify as the geometry of Maritain's real or extra-mental space. The geometric theory of external reality does indeed abstract from a large class of physical properties in the sense of being the metrical study of the coincidence behavior of transported solids independently of the solid's substance-specific physical properties. *But this kind of abstracting does not deprive metrical coincidence behavior of its physicality.*¹³

The key to this argument is Grünbaum's claim that Maritain's view of abstraction disqualifies the application of abstract Euclidean objects to real space, for such objects are real only in the imagination. Grünbaum claims that geometry is applied to external reality without abstracting from the metrical coincidence behavior of its physical characteristics. In contrast, Grünbaum

Grünbaum suffered the effects of a poor earlier translation. However, a comparison of both translations with the French does not reveal a fatal flaw. More germane is this observation made by Wallace: "Throughout Grünbaum's discussion, however, no appraisal or critique is given of the doctrine of abstraction, on which Maritain's statement is clearly based, nor, throughout the book, is there any recognition by the author that space may be treated differently by the geometrician than it is by the physicist. The obscurities of Maritain's presentation notwithstanding, there is little profundity in Grünbaum's analysis and rebuttal. Or, to put it in another way, the basic presuppositions of the two authors are so different that they almost preclude any intelligible discourse between them." Ibid.

¹³ Ibid., 151 (emphasis added).

claims, Maritain's notion of abstraction evacuates the geometrical of its metrical character. This deprives the philosopher of his *a priori* claim to intuit the metric of physical space.

III. Maritain's Views

Let us now turn to examine Maritain to see if Grünbaum's case is complete. First, Maritain's assessment hinges upon a threefold distinction about the reality of the notion of space from the perspectives of the geometrician, the physicist, and the philosopher. This distinction allows Maritain, in the second place, to advance a dilemma to establish the unreality of non-Euclidean space.

Maritain's threefold distinction between the "real space" of the mathematician, the physicist, and the philosopher is, in his mind, of decisive importance. By contrast, Grünbaum appears puzzled by the terminology, and he considers the distinction between the last two "an empty distinction without a difference."¹⁴ The mathematician's space is that coherent object constructed from the axioms of geometry; in this sense, Euclidean and non-Euclidean spaces are "equally 'real'" and "equally true."¹⁵ The physicist's space is whatever space as coordinate system underlies all events when matching mathematical models to matter. By contrast, the philosopher's space is that space which, conceivable by the mind, exists also outside the mind, "not, doubtless, under the conditions proper to mathematical abstraction, but insofar as its definition reveals in a pure state or according to its ideal perfection such or such characteristics

¹⁴ Ibid., 148.

¹⁵ Jacques Maritain, *Distinguish to Unite, or The Degrees of Knowledge*, edited by Ralph M. McInerny, translated by Gerald B. Phelan, *The Collected Works of Jacques Maritain*, vol. 7 (Notre Dame, Indiana: University of Notre Dame Press, 1995), 176.

(pertaining to *the accident of quantity*) which exist or can exist in the world of bodies.”¹⁶ Hence, Grünbaum’s puzzlement: the philosopher’s space appears, to him, to be that mathematical space applicable to physical reality. Is this not also the physicist’s “real space”? Perhaps Maritain’s further sub-distinction that in some other sense “real space” is physical and non-Euclidean when qualified by material objects added to Grünbaum’s confusion.¹⁷ By this further distinction, however, Maritain merely means to motivate his later point that the philosopher makes use of the tools that modern physics offers him.¹⁸ Yet it is still the case, avers Maritain, that the philosopher is not required to abandon the reality of Euclidean space.¹⁹

Maritain’s destructive dilemma by which he argues for a Euclidean metric for real space is founded on a two-part criterion:

We may either analyze the genesis of the notions in order to see if the entity in question, without involving any internal contradiction or impossibility in its constitutive notes . . . , does not imply a condition impossible with existence outside the mind. . . . Or we may consider a condition to which the philosopher knows that the reality of mathematical entities is subject. (He knows that for these entities to exist outside the mind means to exist with sensible existence, and that whatever cannot be constructed in imaginative intuition, which represents freely and in a pure fashion whatever belongs to quantity, has *a fortiori* no possibility of being posited in sensible existence.) This condition is direct

¹⁶ Ibid., 177; italics added.

¹⁷ Ibid., 180–81.

¹⁸ The later point is at *ibid.*, 194–95; see section 8 below.

¹⁹ Ibid., 180n55.

constructibility in intuition.²⁰

Maritain then argues, as Grünbaum noted, that neither option produces the desired result.

Non-Euclidean spaces are not directly constructible in the imagination.²¹ Furthermore, the genesis of non-Euclidean geometric objects must first use Euclidean objects abstracted from sense experience and these by analogy are used to construct non-Euclidean geometries.

Crucially, what modern mathematics takes to be a mere generalization of geometric concepts, Maritain takes to be a logical transfer of meanings to construct new (non-Euclidean) geometric relationships.²² Maritain's Thomistic view requires abstraction-with-analogical construction to arrive at non-Euclidean objects, while Grünbaum's modern view utilizes mathematical generalization.

This difference marks their equivocal understandings of the abstraction which characterizes the difference between physical and mathematical objects. To Maritain, non-Euclidean geometric objects are abstract in comparison to physical magnitude in a different

²⁰ Ibid., 178–79. By “impossibility” Maritain might have in mind something along the lines of Leibniz's notion of impossibility. However, given that he is arguing from incoherence to real impossibility, he probably has in mind the idea that the *rationes* or “notes” of our idea of an essence cannot imply a contradiction in the essence itself, which would imply an existing self-contradiction at the level of being; see St. Thomas, *Summa Theologiae*, Ia, q. 25, a. 3, c. See also James F. Ross, *Thought and World: The Hidden Necessities* (Notre Dame, IN: University of Notre Dame Press, 2008), 21–43.

²¹ Maritain, *The Degrees of Knowledge*, 58 and 153.

²² Ibid., 179–80, 58.

way than are Euclidean objects. To Grünbaum, the abstractions are of a piece and merely differ by degree of generalization (i.e., Euclidean geometry is a special case of Riemannian 3-space). In light of this, is Grünbaum's critique misplaced? It would seem so: Grünbaum has not refuted Maritain but only contradicted him. It also explains, at the very least, why it is that for Grünbaum a single decision procedure about physical metric congruence suffices to decide which geometry of all the possible ones is the geometry of real space, whereas, for Maritain, it is impossible for such a procedure to work. Therefore, if there is a way to show the unsoundness in Maritain's use of the Thomistic doctrine of abstraction while saving the concept of physical metric congruity, then such a critique of Maritain would be more precise than Grünbaum's.

IV. De Koninck's Critique

In his 1934 doctoral dissertation on the philosophy of Arthur Eddington, De Koninck begins his attack on Maritain's contention "that real space is necessarily tridimensionally Euclidean."²³ He notes the same weaknesses of Maritain's use of intuition and logical coherence to refute non-Euclidean geometries as did Grünbaum. However, unlike Grünbaum, De Koninck fixes his sights upon the two-part criterion grounding Maritain's destructive dilemma, which he claims causes two errors. Firstly, Maritain confuses *extension* with *quantity*. The first confusion allows Maritain to import a Euclidean metric into his notion of philosophically real space, and this causes Maritain's second error (which in turn prevents him from correcting his first one). That is, secondly, Maritain does not grasp the true nature of the act of measurement. To the

²³ Charles De Koninck, "The Philosophy of Sir Arthur Eddington," in *The Writings of Charles De Koninck: Volume One*, edited and translated by Ralph McInerny (Notre Dame, Indiana: University of Notre Dame Press, 2008), 147.

contrary, “As a philosopher, [Maritain] can say nothing about the metric structure of space. That is for the physicist. And he replies that there is curved space.”²⁴ Because this claim about the actual metric of space is only justified by the nature of measurement and hence the character of the formal object of physics, De Koninck claims: “[Maritain’s] whole philosophy of science is thereby vitiated.”²⁵ Let us examine each of the two errors De Koninck ascribes to Maritain in the next two sections.

V. Extension and Quantity

Maritain’s first error, claims the younger Thomist, lies in his confusing *extension* and *quantity*. Maritain writes:

When we consider things from the point of view of the philosopher and not of the physicist, and speak the former’s language, then quantity, *that is to say the extension of substance and of its metaphysical unity into diverse parts according to position*, is a real property of bodies. There are in nature real dimensions, numbers and measurements, a real space, a real time. It is precisely under the conditions and modalities of this real quantity, or, to put it in another way, it is as quantitatively measured and regulated, that the interacting causes in nature develop their qualitative activities.²⁶

In this passage, Maritain anticipates his threefold distinction between mathematical, physical, and philosophical space. The portion italicized above, De Koninck claims, is the error.

²⁴ Ibid., 149. Cf. Grünbaum, *Philosophical Problems of Space and Time*, 148.

²⁵ De Koninck, “The Philosophy of Sir Arthur Eddington,” in *The Writings of Charles De Koninck*, 154.

²⁶ Maritain, *The Degrees of Knowledge*, 150–52; italics added.

Maritain unites what should be distinguished, extension and quantity, or so claims De Koninck.²⁷ Briefly, extension pertains to substances, quantity to accidents. The extension or exteriority of a substance grants it parts outside of parts in an indeterminate, negative relation: *this* part is not *that* part. Nor is there any qualitative differentiation in this opposition; the opposition between this part and that part is homogenous or material (and hence is the principle of individuation).²⁸ Quantity, by contrast, is what is made known by measurement and provides parts with a determinate relationship towards one another as accidents of individual substances. To the mathematician, homogeneous exteriority is analogous to the modern geometric manifold that possesses quantity as a metric feature; to the physicist, a metric is quantity calculated upon homogeneous exteriority via measurement. Hence, confusing extension with quantity

²⁷ See De Koninck, “The Philosophy of Sir Arthur Eddington,” in *The Writings of Charles De Koninck*, 148 and 148n66 (see also 224–25, 259, 294, 425).

²⁸ The complexities of the Thomistic doctrine on the principle of individuation cannot be discussed here; the *locus classicus* is the fourth question of St. Thomas Aquinas, *Faith, Reason, and Theology: Questions I-IV of His Commentary on the “De Trinitate” of Boethius*, translated by Armand Maurer (Toronto: The Pontifical Institute of Mediaeval Studies, 1987); see also John F. Wippel, *The Metaphysical Thought of Thomas Aquinas: From Finite Being to Uncreated Being* (Washington, D.C.: The Catholic University of America Press, 2000), 351–77. An interesting proposal applying this doctrine in tandem with Big Bang cosmology can be found in Travis Dumsday, “Why Thomistic Philosophy of Nature Implies (Something Like) Big-Bang Cosmology,” *Proceedings of the American Catholic Philosophical Association* 85 (2011): 69–78.

surreptitiously introduces a metric into one's philosophical conception of real space.

De Koninck locates the root of this first error in the two-part criterion driving Maritain's destructive dilemma. It made "sensible existence" the criterion for what is possible in material reality. Rather, "[Maritain] should have said 'material existence'" because "not everything that is material is sensible. To elevate sensibility into a criterion of material reality is a restriction of it to what can be imaginatively represented."²⁹ Here De Koninck critiques Maritain on properly Thomistic grounds. Maritain does not espouse the error generally, of course.³⁰ Yet even Homer nods. The depths of matter as a cause are not knowable by direct sensible observation. De Koninck offers as examples the primary matter of substances and the quantum structure of atoms, both of which are scarcely intelligible to mind, even by various representative analogates. The metric structure of physical space, due to its deep ties to materiality, is thus a candidate for a reality whose character might be other than its mathematical counterpart. Maritain's is a quasi-Platonic error about matter.³¹

VI. The Nature of Measurement

Maritain's first error causes the second: "No doubt the [philosophically] real space of Maritain is metric. How has he measured it? That is the whole problem."³² Maritain has not

²⁹ Ibid., 148.

³⁰ See Maritain, *The Degrees of Knowledge*, 187.

³¹ See Aristotle, *Physics*, 1.8, 191b35.

³² De Koninck, "The Philosophy of Sir Arthur Eddington," in *The Writings of Charles De Koninck*, 149.

sufficiently considered the nature of measurements that make known physical magnitudes.³³

Here we must note that De Koninck does not address Maritain's scintillatingly opaque footnote that sketches a Thomistic philosophy of measurement based upon the doctrines of categorical, rational, and transcendental relations found in Aristotle, Thomas Aquinas, and the seventeenth-century Thomistic commentator John Poinsoot.³⁴ First, we will assume that De Koninck thought his critique was sufficient before addressing its limitations (see section 8 below).

In order to discover a physical magnitude, one must select a unit of measure:

Is there in things an absolute corresponding to length? The enumeration that enables us to attain a pure number is an absolute operation. Length is not a pure number, it is a physical magnitude. Its definition resides in the description of the process of

³³ De Koninck follows the views of Fernand Renoirte, his dissertation director. Fernand Renoirte, "La théorie physique. Introduction à l'étude d'Einstein," *Revue Néo-Scholastique de Philosophie* 25, no. 100 (1923): 349–75; and Renoirte, "La critique Einsteinienne des mesures d'espace et de temps," *Revue Néo-Scholastique de Philosophie* 26, no. 3 (1924): 267–98. These are a scholastic adaptation of what Grünbaum would call Poincaré's empirical conventionalism. See Henri Poincaré, *The Value of Science: The Essential Writings of Henri Poincaré*, edited by Stephen Jay Gould, translated by George Bruce Halsted and Francis Maitland (New York: Modern Library, 2001); see also Grünbaum, *Philosophical Problems of Space and Time*, 115–131.

³⁴ Maritain, *The Degrees of Knowledge*, 151n13.

measurement *which includes an instrument one can only show*.³⁵

On this account, length for the physicist is inextricable from a system of references defining the process of measurement. In this sense, physical magnitude is not an abstract idea but the idea of extension brought to a concrete standard (individualized, here and now) grounding a process of measurement whose result is a known quantity: physical magnitude. This is the *formal object* of physics, the intellectual means it uses to know its object of study.³⁶ By contrast, to claim that lengths are “absolute” or “determinate in themselves” can quickly lead to tautologies. “A length is always the same as itself” is a metaphysician’s identity, but the physicist requires a standard of measure and a convention about rigid rods—particular objects here and now—undergoing transport and comparisons to distant objects, not merely a self-identity obtaining at one place and time. The “absolute” quantities of length or time in the cosmos, which Maritain’s “philosopher” and his “pure spirits” know to exist but do not know how to measure, equivocate on the issue of what a physical magnitude is.³⁷ Maritain defines a physical magnitude as “absolute” or determinate in itself apart from how it is measured. De Koninck claims that a physical magnitude has meaning only if one specifies how it is measured. Indeed, Maritain’s absolute lengths exist in a closed and thus unknowable system. He is not interested, claims De Koninck, in defining a process of measurement to manifest the metric of these absolute lengths because he has already

³⁵ De Koninck, “The Philosophy of Sir Arthur Eddington,” in *The Writings of Charles De Koninck*, 149.

³⁶ *Ibid.*, 154–58.

³⁷ See Maritain, *The Degrees of Knowledge*, 167–68.

conflated extension with quantity.³⁸

VII. The Critiques Compared

We noted above that if there were a way to show the unsoundness in Maritain's use of the Thomistic doctrine of abstraction while preserving the necessary concept of physical metric congruity, then such a critique of Maritain would be more precise than Grünbaum's. However, De Koninck seems to have succeeded on both counts. First, his explanation of the process of measurement as the means by which to isolate the metric of real space meets Grünbaum's needs. Second, De Koninck points out how Maritain was mistaken in his employment of abstraction in the account of physical magnitude. To see this second point more clearly, we must consider the Thomistic doctrine of abstraction in the next two sections.

VIII. Shadows and Symbols

First, a small gloss on two glosses by way of preparation. De Koninck and Maritain give opposing reasons for the truth of Eddington's remark: "The external world of physics has become a world of shadows."³⁹ For Maritain, the world of physics is a shadow-world in comparison to "the universe with which we are familiar." Maritain's philosopher knows that the physicist's use of symbols as beings of reason are "so many points of emergence through which an aspect of things existing in themselves appear to us."⁴⁰ On this point, the younger Thomist did

³⁸ De Koninck, "The Philosophy of Sir Arthur Eddington," in *The Writings of Charles De Koninck*, 152, 153.

³⁹ Sir Arthur S. Eddington, *The Nature of the Physical World* (Ann Arbor, Michigan: Ann Arbor Paperbacks, 1963), xvi.

⁴⁰ Maritain, *The Degrees of Knowledge*, 170.

not fully appreciate Maritain's grasp of the nature and use of symbols in physics, even if he would later learn from its nuance to clarify his own views. Maritain sees physics as defined by a tendency towards what is mathematically intelligible, which tendency issues in the construction of beings of reason. Hence, truths about the physical world are "mathematically recast into the very heart of geometry itself"⁴¹ and this modern, algebraic geometry is used as a symbol of what is physically real. Consequently, these beings of reason are used to tell "the well-founded myths of science" which the philosopher—operating in his essentially distinct realm of intelligibility—can only understand by proposing corresponding interpretive myths of his own.⁴²

Contrary to this dualism of myths, the younger De Koninck sees modern physics as defined by a tendency towards understanding what exists in matter insofar as it is knowable to the measuring mind. When the physicist symbolically codifies this measurement, the symbols are not pure symbol—abstract mathematical magnitudes—but in the mind's eye they "move against an obscure backdrop which is the order of non-intuited yet quite real essences. . . . [I]t is this background that gives a meaning to symbols."⁴³ Only later in his career did De Koninck realize, as Maritain already had, that *this very stance of the mind* towards symbols as tools implicates beings of reason in the physicist's act of measurement. Nonetheless, their contrary interpretations of symbols is clear. Maritain's view grants physics a knowledge of the real *only in* its symbols,

⁴¹ Ibid., 184.

⁴² Ibid., 194–95.

⁴³ De Koninck, "The Philosophy of Sir Arthur Eddington," in *The Writings of Charles De Koninck*, 212.

while De Koninck's view claims for physics a knowledge of the real *through* its symbols.⁴⁴ The significance of this slight difference requires an examination of the Thomistic answer to the central philosophical question about mathematical physics (section 1 above).

IX. Thomistic Abstraction

The Thomistic doctrine of abstraction is presented here in thumbnail sketch.⁴⁵ Generally, Thomists mean by "abstraction" the consideration of one thing without another. Metaphysical abstraction (more properly, "separation") is not at stake here. Rather, the key difference lies

⁴⁴ For his more mature views, see Charles De Koninck, "Introduction à l'étude de l'âme," *Laval théologique et philosophique* 3, no. 1 (1947): 9–65; "Abstraction from Matter: Notes on St. Thomas's Prologue to the 'Physics,'" *Laval théologique et philosophique* 13 and 16 (1957): 133–96; 53–69, 169–88. See also John G. Brungardt, "Charles De Koninck and the Sapiential Character of Natural Philosophy," *American Catholic Philosophical Quarterly* 90, no. 1 (2016): 1–24, as well as John G. Brungardt, "St. Thomas and Modern Natural Science: Reconsidering Abstraction from Matter," in *Cognoscens in Actu Est Ipsum Cognitum in Actu: Sobre Los Tipos y Grados de Conocimiento*, ed. Carlos Augusto Casanova G. and Ignacio Serrano del Pozo (Santiago: RIL Editores, 2018), 433–71.

⁴⁵ For details see St. Thomas Aquinas, *The Division and Methods and the Sciences: Questions V and VI of His Commentary on the "De Trinitate" of Boethius*, 2nd ed., translated by Armand Maurer (Toronto: The Pontifical Institute of Mediaeval Studies, 1958). See also John F. Wippel, *The Metaphysical Thought of Thomas Aquinas: From Finite Being to Uncreated Being* (Washington, D.C.: The Catholic University of America Press, 2000), 3–62; Maritain, *The Degrees of Knowledge*, 37–72; and De Koninck, "Abstraction from Matter."

between the first and second degrees of abstraction. In the first degree of abstraction the mind considers physical realities without their particularity yet retains, in its conception, their materiality; this results in universal thoughts about things. Natural philosophy (what Aristotle called “physics”) operates within this first degree. In the second degree of abstraction the mind considers certain intelligible features of reality which can be understood without any change or materiality whatsoever. This is the abstraction used by mathematics, which considers quantities and certain of their various qualitative features and relations (e.g., their shapes and ratios) insofar as they are unchanging. Thus, abstract physical objects are opposed in their very constitution to abstract mathematical objects, for the first retains while the second eliminates reference to materiality as the necessary condition of change.

For Aquinas, physical objects and mathematical objects are each prior to the other, but in different ways. In being, the physical object is prior, and hence the abstract physical object is genetically prior in the mind to the abstract mathematical object. However, in formal simplicity and thus in intelligibility, the mathematical object is prior to the abstract physical object. This twofold priority shows itself in the contrast between the Thomistic and modern approaches to explain the continuum. Aquinas argues that natural philosophy demonstrates the existence of the continuum employed by geometry.⁴⁶ In turn, discrete quantities (numbers) are abstracted from

⁴⁶ See St. Thomas Aquinas, *Commentary on Aristotle's Posterior Analytics*, translated by Richard Berquist (South Bend, Indiana: St. Augustine's Dumb Ox Books, 2008), book I, lectio 5; Anne Newstead, “Aristotle and Modern Mathematical Theories of the Continuum,” in *Aristotle and Contemporary Philosophy of Science*, edited by D. Sfendoni-Mentzou, J. Hattiangadi, and D. M. Johnson, vol. 2 (Frankfurt: Peter Lang, 2001), 113–29. See also Maritain, *Degrees of*

the divisible (that is, countable) geometric continuum. The modern approach to the continuum, exemplified by Dedekind, inverts this order and places the discrete ahead of the continuous, *whether geometrically or physically conceived*.⁴⁷ Indeed, Grünbaum noted as much against Maritain; the priority of the real number system to the geometric continuum permits the moderns to prove the consistency of Euclidean and non-Euclidean geometries.

Thus, when approaching the problem of the physical continuum, the *via moderna* utilizes metrics that can be allowed to vary as functions; this method anticipates all possible metrics obtaining in real space. On the Thomistic view, however, matter (the root of variability and changeability) is removed in the second degree of abstraction; hence, its metric is Euclidean before anything else. Yet *if materiality can cause metric variability*, this requires one to admit that the metric structure of physical quantity can only be known in material objects by empirical discovery. Indeed, the presence of matter permits the possibility that the physical metric of space behaves in a way that is different from a Euclidean metric which has been evacuated of all potency for change. Thus, finding the metric of physical space requires one to join the second degree of abstraction to the first insofar as this is possible. This is done through the act of measurement, which is both physical and mental. De Koninck is correct about the former: this

Knowledge, 43–44n30, as well as De Koninck, “The Philosophy of Sir Arthur Eddington,” in *The Writings of Charles De Koninck*, 169–71.

⁴⁷ For a concise history of this reversal, see Tim Maudlin, *New Foundations for Physical Geometry: The Theory of Linear Structures* (Oxford: Oxford University Press, 2014), 6–25. See also Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (Mineola, New York: Dover Publications, 1992).

“joining” requires an account of measurement with a referentially defined measurement standard if we wish to know the metric of the physical cosmos. Maritain is correct about the latter: insofar as this “joining” by mind unites in its consideration two types of abstraction which are defined in opposition to each other, the physicist must utilize a being of reason to achieve his ends.

X. Conclusion

De Koninck’s critique is superior to Grünbaum’s insofar as it meets the requirements of refutation and not mere contradiction. Grünbaum’s critique of Maritain’s view of abstraction fails to reach his opponent due to an equivocation. De Koninck then shows how Maritain misapplies Thomistic abstraction doctrine in a twofold error regarding quantity and measurement. Consequently, De Koninck’s account is a better refutation and thus better able to help the Thomist navigate the ontology of physical space. How might this help establish a more adequate account of the cosmos? A more adequate ontology is perhaps available to the Thomistic view precisely insofar as it places a priority upon substance, space and time, in its consideration of changeable being. However, that discussion is beyond the scope of this essay.⁴⁸

⁴⁸ The author thanks the audience members at the 2018 American Maritain Association meeting for their comments, as well as Jack Cahalan, Thomas McLaughlin, Matthew Miner, Martin Beers, Oscar Leon, and José Tomás Alvarado for their various conversations on this and related topics. The essay was written as a part of my postdoctoral research at Pontificia Universidad Católica de Chile, funded by FONDECYT, Postdoctoral Project No. 3170446. I am grateful to both for their support.

